# TRINITY COLLEGE

# Mathematics Methods U 3,4
# Test 2 2022

Section 1 Calculator Free

**Applications of Antidifferentiation, Exponential and Trigonometric Functions**

**STUDENT'S NAME** ______________________________

**DATE**: Monday 4th April **TIME:** 35 minutes **MARKS**: 35

**INSTRUCTIONS:**

Standard Items: Pens, pencils, drawing templates, eraser, formula sheet

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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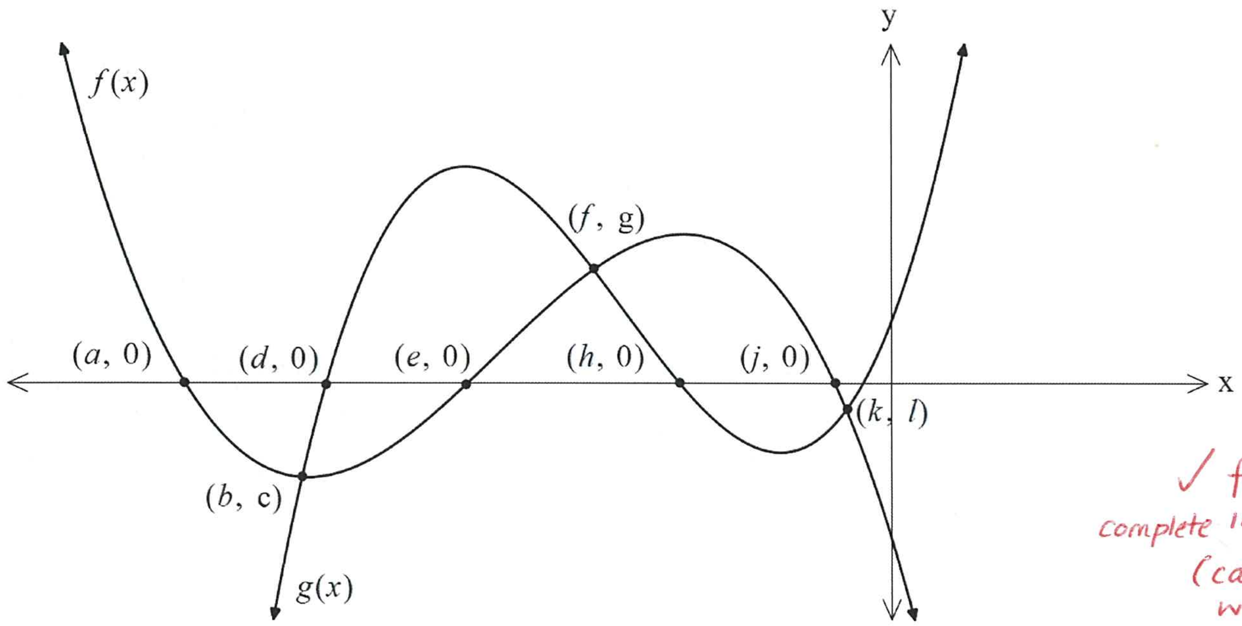
1. (2 marks)

Write an **expression** for the area enclosed between the functions $f(x)$ and $g(x)$.




✓ for each complete integral (can be written as individual integrals)

$$\int_b^f (g(x)) - f(x)\,dx + \int_f^k f(x) - g(x)\,dx$$

✓ ✓

2. (8 marks)

Find $\frac{dy}{dx}$ for the following:

(a) $y = 5e^{3x}$

$\frac{dy}{dx} = 15e^{3x}$ ✓

[1]

(b) $y = \frac{e^{5x} + e^{x}}{e^{3x}}$ (*Hint: Simplify the expression first*) [2]

$y = e^{2x} + \frac{1}{e^{2x}}$

$\frac{dy}{dx} = 2e^{2x} - \frac{2}{e^{2x}}$

✓ for simplifying

✓ for correct derivative from simplification

(c) $y = x^3 e^{x^2+1}$ (*Do NOT simplify your answer*) [3]

$u = x^3$ $v = e^{x^2+1}$

$u' = 3x^2$ $v' = 2x\,e^{x^2+1}$

$\frac{dy}{dx} = 2x^4 e^{x^2+1} + 3x^2 e^{x^2+1}$

✓ for derivative u

✓ for derivative v

✓ for product rule

(d) $y = (1 - \cos x)^3$ [2]

$\frac{dy}{dx} = 3(1 - \cos x)^2 (\sin x)$ ✓ ✓

✓ for chain rule

✓ for derivative of $1 - \cos x$

3. (9 marks)

Evaluate each of the following definite integrals, leaving your answers as exact values:

(a) $\int_0^1 \frac{3x^2}{4} + \sqrt{x} - 2\,dx$ [3]

$= \left[ \frac{x^3}{4} + \frac{2}{3}x^{3/2} - 2x \right]_0^1$

$= \left( \frac{1}{4} + \frac{2}{3} - 2 \right)$

$= \frac{3}{12} + \frac{8}{12} - \frac{24}{12} = \frac{-13}{12}$

✓ for integral

✓ for sub $x = 1$

✓ for answer

(b) $\int_0^1 (1 + e^{2x})\ dx$ [3]

$= \left[ x + \frac{1}{2}e^{2x} \right]_0^1$

$= \left(1 + \frac{1}{2}e^2\right) - \left(\frac{1}{2}\right)$

$= \frac{1}{2} + \frac{e^2}{2} \quad \left( \frac{e^2 + 1}{2} \right)$

✓ for integral

✓ for sub $x = 1$, $x = 0$

✓ for answer

(c) $\frac{d}{dx}\left(\int_1^x (3t^2 - 5)dt\right) = 3x^2 - 5$ [1]

✓ for F.T.C

(d) $\int_1^2 \frac{d}{dx}\left(\frac{1+x}{1+x^2}\right)dx = \left[ \frac{1+x}{1+x^2} \right]_1^2$ [2]

$= \left(\frac{3}{5}\right) - \left(\frac{2}{2}\right)$

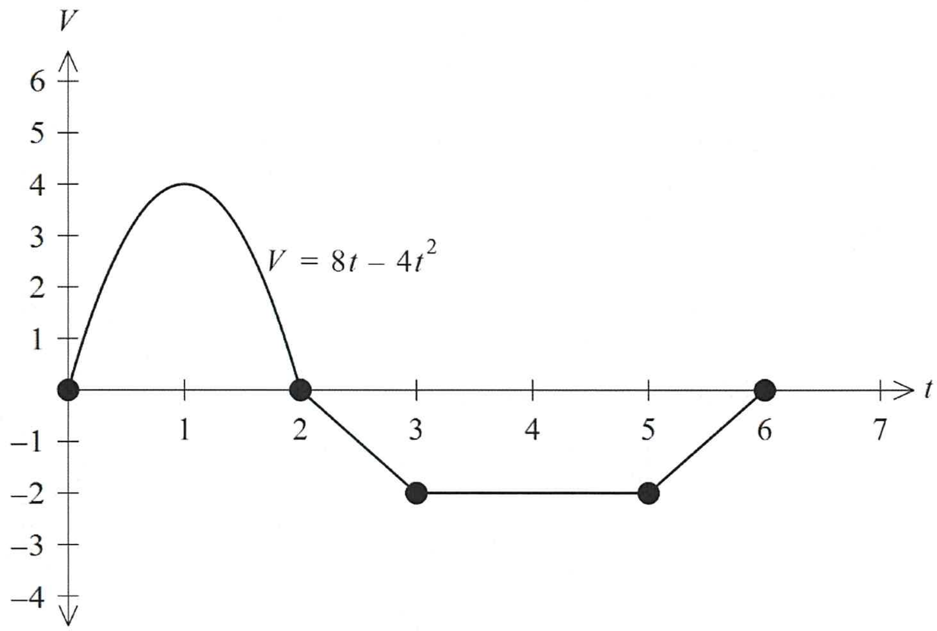
$= -\frac{2}{5}$

✓ for correct function

✓ for sub $x = 2$, $x = 1$

4. (6 marks)

The graph below represents a velocity – time graph of a body moving in rectilinear motion, where V is measured in metres per second.




(a) Find the distance travelled in the first two seconds. [2]

$$\int_0^2 8t - 4t^2 \, dt = \left[4t^2 - \frac{4}{3}t^3\right]_0^2$$

✓ for integral

$$= \left(16 - \frac{32}{3}\right) - 0$$

✓ for sub $x = 2$, $x = 0$

$$= \frac{16}{3} \text{ m}$$

(b) Find the distance travelled in the first five seconds. [2]

$$\int_0^2 8t - 4t^2 \, dt + \frac{1}{2}(1)(2) + (2 \times 2)$$

$$= \frac{16}{3} + 1 + 4$$

$$= \frac{31}{3} \text{ m}$$

✓ for correct area $2 \to 5$ (Trapezium OR △ + □

✓ for total area

(c) Find the distance from the starting point after five seconds. [2]

$$= \frac{16}{3} + (-1) + (-4)$$

✓ for restating areas as −ve

$$= \frac{1}{3} \text{ m from starting point}$$

✓ for total area

5. (7 marks)

The function $y = f(x)$ is continuous for all real values of $x$ and $f(x) \geq 0$ for $1 \leq x \leq 4$.

It is known that $\int_1^4 f(x)dx = A$ and $\int_4^6 f(x)dx = -B$ where $A$ and $B$ are positive real numbers.

Find, with reasons, in terms of $A$ and/or $B$ where appropriate:

(a) the area of the region trapped between the curve $y = 2f(x)$, the $x$-axis and the lines $x = 1$ and $x = 4$. [2]

$$\int_1^4 2f(x) = 2\int_1^4 f(x)\,dx = 2A$$

✓ for scalar multiple

✓ for 2A

(b) $\int_1^6 f(x)\,dx$ [2]

$$\int_1^4 f(x)\,dx + \int_4^6 f(x)\,dx = A - B$$

✓ for splitting integrals

✓ for A − B

(c) $\int_4^6 2x - f(x)\,dx$ [3]

$$\int_4^6 2x\,dx - \int_4^6 f(x)\,dx = \left[x^2\right]_4^6 - (-B)$$

$$= 36 - 16 + B$$

$$= 20 + B$$

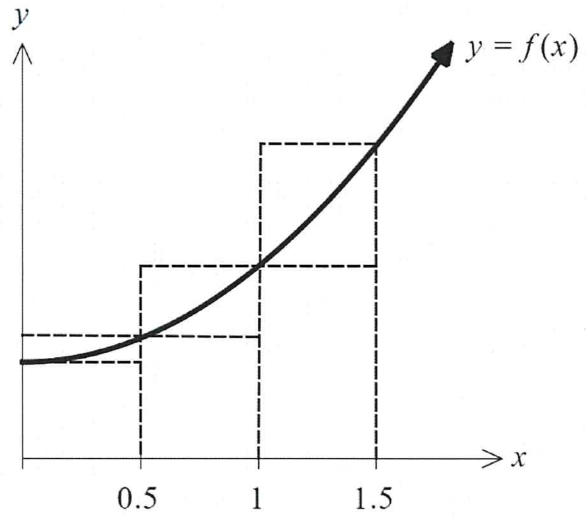
✓ for splitting integrals

✓ for evaluating $\left[x^2\right]_4^6$

✓ for 20 + B

6. (3 marks)

Consider the function $y = f(x)$ shown graphed below. The table gives the value of the function at the given $x$ values.




✓ for UNDER

✓ for OVER

✓ for stating $\int_0^{1.5} f(x) =$ exact area

| $x$ | 0 | 0.5 | 1 | 1.5 |
|---|---|---|---|---|
| $f(x)$ | 20 | 21 | 24 | 29 |

By considering the areas of the rectangles shown, demonstrate and explain why

$32.5 < \int_0^{1.5} f(x)\,dx < 37$.

Under Estimate $= (20 \times 0.5) + (21 \times 0.5) + (24 \times 0.5)$

$= 10 + 10.5 + 12$

$= 32.5$

Over Estimate $= (21 \times 0.5) + (24 \times 0.5) + (29 \times 0.5)$

$= 10.5 + 12 + 14.5$

$= 37$

$\int_0^{1.5} f(x)\,dx =$ exact area

$\therefore\ 32.5 < \int_0^{1.5} f(x)\,dx < 37$

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# Mathematics Methods Unit 3,4
# Test 2 2022

## Section 2 Calculator Assumed
**Applications of Antidifferentiation, Exponential and Trigonometric Functions**

**STUDENT'S NAME** ______________________________

**DATE**: Monday $4^{th}$ April **TIME:** 15 minutes **MARKS**: 14

**INSTRUCTIONS:**

Standard Items: Pens, pencils, drawing templates, eraser, formula sheet
Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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5. (6 marks)

The rate of change of concentration of a pollutant in a water reservoir can be expressed by $\frac{dC}{dt} = kC$, where $C$ is the concentration in parts per million $t$ days after observations began and $k$ is a constant.

The initial concentration of the pollutant was 82 ppm. Two weeks later this value had dropped to 35 ppm.

(a) Show that the value of k is -0.0608. ~~[3]~~ 2

$C_0 = 82$ $\quad t = 14 \quad C = 35\text{ ppm}$

$C = C_0 e^{kt}$

$35 = 82 e^{14k}$

(use CAS solve for $k$)

$k = -0.0608$

✓ for $C_0 = 82$

✓ for using $(14, 35)$

(b) Determine the concentration of the pollutant after three weeks. ~~[1]~~ 2

$C = 82 e^{-0.0608(21)}$

$\approx 22.8721$ ppm

$\approx 23$ ppm

✓ for sub $t = 21$

✓ for answer

(c) The water can be used for drinking once the concentration of the pollutant falls below 5 parts per million.
Determine how long it will take for the concentration to reach this level. [2]

$5 = 82 e^{-0.0608(t)}$

✓ for equation $= 5$

(use CAS solve for $t$)

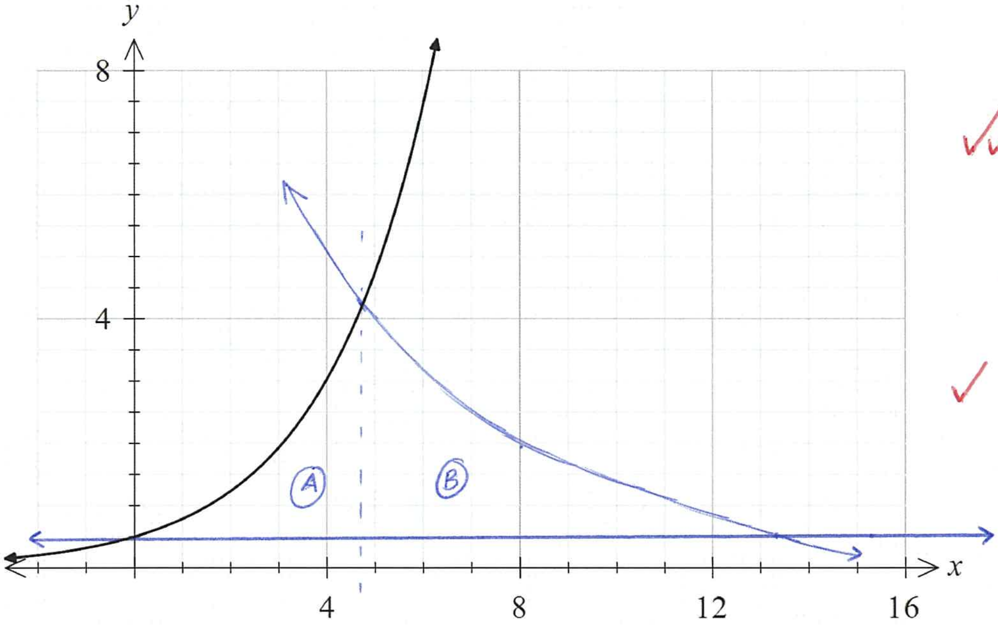
$t \approx 46.01$

$\approx 46$ days

✓ for solving for $t$

6. (8 marks)

Three functions are defined by $f(x) = 14e^{-0.25x}$, $g(x) = 0.5e^{0.45x}$ and $h(x) = 0.5$.




✓✓ for $14e^{-0.25x}$
✓ for curve
✓ for accuracy (p. of Int)
✓ for $h(x)$

(a) One of the functions is shown on the graph above. Add the graphs of the other two functions. [3]

(b) Working to three decimal places throughout, determine the area of the region enclosed by all three functions. [5]

$f(x) = g(x)$

$x = 4.760$

$$A = \int_0^{4.760} 0.5e^{0.45x} - 0.5 \, dx$$

$$= 5.972$$

$f(x) = h(x)$

$x = 13.329$

$$B = \int_{4.760}^{13.329} 14e^{-0.25x} - 0.5 \, dx$$

$g(x) = h(x)$

$x = 0$

$$= 10.752$$

✓ for $f(x) = g(x)$

✓ for $f(x) = h(x)$

✓ Area A (using p. of I found)

✓ Area B (using P. of I found)

✓ Total Area A + B

Area = A + B

$= 16.724$ units$^2$